

# $0^+$ and $1^+$ States of $B$ and $B_s$ Mesons

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Predicted masses of  $J^P = 0^+$  and  $1^+$  states of  $D_{sJ}$  and  $D$  by a potential model proposed some time ago by two of us (T.M. and T.M.), in which the Hamiltonian and wave functions are expanded in  $1/m_Q$  with  $m_Q$  heavy quark mass respecting heavy quark symmetry, have recently been confirmed by BaBar and Belle experiments within one percent accuracy.

In this Letter, decay modes of  $0^+$  and  $1^+$  states of  $B$  and  $B_s$  mesons are discussed using the predicted masses of this model.

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## I. INTRODUCTION

The BaBar's discovery [1] of  $D_{sJ}(2317)$  and  $D_{sJ}(2460)$  of the system  $c$  and  $s$  quarks has inspired theorists [2, 3] to explain these states in some way because a well-known potential model [4, 5] failed to expound masses of these states. This discovery has been soon confirmed by CLEO and Belle [6]. These  $D_{sJ}$  states are narrow because isospin violation and  $DK/D^*K$  threshold prohibit them to decay into lower states  $D_s(1968)$  and  $D_s^*(2112)$ . This is another reason why theorists are interested in these states and are trying to explain their branching ratios, too.

Soon after the discovery, another set of heavy mesons,  $D_0^{*0}(2308)$  and  $D_1^{*0}(2427)$  of  $c$  and  $u/d$  quarks which have the same quantum numbers  $J^P = 0^+$  and  $1^+$  as  $D_{sJ}$ , has been discovered by Belle [7]. Again masses of these states cannot be fitted with those of a potential model [4, 5]. These states  $D_0^{*0}$  and  $D_1^{*0}$  have no restriction like  $D_{sJ}$  and hence their decay (like  $D\pi$  and  $D^*\pi$ ) width becomes broad.

To explain masses of  $D_{sJ}$ , Bardeen, Eichten, Hill, and others [2, 3] proposed an interesting idea of an effective Lagrangian with chiral symmetries of light quarks and heavy quark symmetry. The heavy meson states with the total angular momentum  $J = 0$  and  $J = 1$  with  $j_q = 1/2$ , which is the total angular momentum of the light quark degrees of freedom, make the parity doublets  $(0^-, 0^+)$  and  $(1^-, 1^+)$ , respectively, and the members in each doublet degenerate in a limit of chiral symmetry. Furthermore, the two states  $(0^-, 1^-)$  degenerate in a limit of heavy quark symmetry, as well as  $(0^+, 1^+)$  do. These doublets,  $(0^-, 1^-)$  and  $(0^+, 1^+)$ , are called the heavy spin multiplets. Though their model can reproduce well the mass difference between members in each parity doublet by using the modified Goldberger-Treiman relation, unfortunately it cannot calculate masses themselves. This is why some people have proposed a modified potential model by using a  $DK$  bound state [8].

Several years ago, Godfrey, Isgur, and Kokoski proposed a relativised potential model [4, 5], in which they included  $\sqrt{p^2 + m^2}$  as a kinetic term and spin-dependent interaction terms where linear-rising confining as well as short-range Coulomb potentials are taken into account. This model, however, could not reproduce the masses of  $D_{sJ}(2317)$  and  $D_{sJ}(2460)$ . Even though they have taken an infinite heavy quark mass limit in the second paper [5], they could not reproduce  $D_{sJ}(1^+)$  mass and their calculated masses exceeded  $DK/D^*K$  threshold. Thus, there appear some

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TABLE I: Comparison of higher  $D$  and  $D_s$  meson masses (units in MeV)

$J^P$	$D(0^+)$	$D(1^+)$	$D_s(0^+)$	$D_s(1^+)$
observed	2308	2427	2317	2457
predicted	2304	2449	2339	2487

TABLE II: Higher  $B$  and  $B_s$  meson masses taken from [9] (units in MeV)

$J^P$	$B(0^+)$	$B(1^+)$	$B_s(1^-)$	$B_s(0^+)$	$B_s(1^+)$
predicted	5697	5740	5440	5716	5760

discussions [8] that a potential model is not appropriate to describe these states.

The above potential model does not completely and consistently respect heavy quark symmetry and does not treat quarks as Dirac particles. Some time ago, two of the authors (T.M. and T.M.) [9] proposed a new bound state equation for atomlike mesons, i.e., heavy mesons composed of a heavy quark and a light antiquark. In this model compared with [4, 5], quarks are treated as four-spinor particles from the beginning and both the Hamiltonian and wave functions are expanded in  $1/m_Q$  so that our model treats quarks as relativistic as possible and consistently takes into account heavy quark symmetry within a potential model. Our predicted masses for these states,  $D_{sJ}$  and  $D_0^{*0}$  and  $D_1^{*0}$ , are in good agreement within one percent accuracy when calculated at the first order in  $1/m_Q$ . See Table I in this letter and Tables III and IV in [9].

In this Letter, having confirmed that our model has well succeeded in predicting masses of recently discovered heavy mesons, we predict masses of  $0^+$  and  $1^+$  of  $B$  and  $0^+$ ,  $1^-$  and  $1^+$  of  $B_s$  mesons by citing the evaluated values in [9]. Here we also discuss their decay modes; whether these mesons violate isospin symmetry or not and whether mass difference between  $0^+$  ( $1^+$ ) and  $0^-$  ( $1^-$ ) is less than the  $BK/B^*K$ -mass threshold or not. We expect these higher states of  $B$  and  $B_s$  mesons can be detected in CDF/LHC experiments.

## II. HIGHER $B$ AND $B_s$ MESON MASSES AND THEIR DECAY MODES

Our prediction in [9] of  $B$  and  $B_s$  meson masses in the first order of  $1/m_Q$  corrections is given in Table II. Considering the fact that  $D/D_s$  masses have been well predicted by our model within one percent accuracy as seen in Table I, we expect masses of these  $B$  and  $B_s$  mesons are similarly within one percent accuracy to be observed. Furthermore when looking at Table I carefully, we expect that our predicted masses of  $B/B_s$  mesons agree with experiments better than those of  $D_s$  since  $m_{u,d}/m_b < m_{u,d}/m_c < m_s/m_b < m_s/m_c \ll 1$  ( $m_{u,d} = 10$  MeV,  $m_s = 94.72$  MeV,  $m_c = 1457$  MeV, and  $m_b = 4870$  MeV are adopted in our paper [9]).

Let us discuss decay modes of  $0^+$  and  $1^+$  states of  $B$  and  $B_s$  mesons taking into account these meson masses given by Table II.

- (1)  $B_s(1^-) \rightarrow B_s(0^-) + \gamma$
- (2)  $B(0^+) \rightarrow B(0^-) + \pi$  with broad decay width
- (3)  $B(1^+) \rightarrow B(1^-) + \pi$  with broad decay width
- (4)  $B_s(0^+) \rightarrow B_s(0^-) + \pi$  with narrow decay width
- (5)  $B_s(1^+) \rightarrow B_s(1^-) + \pi$  with narrow decay width

Comments are given as follows.

- (1) Decay  $B_s(1^-) \rightarrow B_s(0^-) + \gamma$  is similar to  $B(1^-) \rightarrow B(0^-) + \gamma$ , i.e.  $B^* \rightarrow B + \gamma$ , and is dominant decay mode.
- (2) Decay width of  $B(0^+) \rightarrow B(0^-) + \pi$  is as broad as a few hundred MeV like  $D(0^+) \rightarrow D(0^-) + \pi$  [7] because this is strong decay and is not prohibited by isospin invariance since  $I(B(0^-)) = I(B(0^+)) = 1/2$  while  $I(\pi) = 1$ , where  $I(X)$  is isospin of a particle  $X$ .
- (3) Decay width of  $B(1^+) \rightarrow B(1^-) + \pi$  is also as broad as a few hundred MeV like  $D(1^+) \rightarrow D(1^-) + \pi$  [7] because this is strong decay and is not prohibited by isospin invariance since  $I(B(1^-)) = I(B(1^+)) = 1/2$  while  $I(\pi) = 1$ .

- (4) Decay width of  $B_s(0^+) \rightarrow B_s(0^-) + \pi$  is expected very narrow, a few MeV like the decay of  $D_s(0^+) \rightarrow D_s(0^-) + \pi$  since this decay mode is prohibited by isospin invariance due to  $I(B_s(0^+)) = I(B_s(0^-)) = 0$  while  $I(\pi) = 1$  and the predicted mass of  $B_s(0^+)$  is below  $BK$  threshold.
- (5) Decay width of  $B_s(1^+) \rightarrow B_s(1^-) + \pi$  is also expected very narrow, a few MeV like the decay of  $D_s(1^+) \rightarrow D_s(1^-) + \pi$  since this decay mode is prohibited by isospin invariance due to  $I(B_s(1^+)) = I(B_s(1^-)) = 0$  while  $I(\pi) = 1$  and the predicted mass of  $B_s(1^+)$  is below  $B^*K$  threshold.

Here  $B_s(0^-) = B_s(5370)$ ,  $D_s(0^-) = D_s^\pm(1968)$ ,  $D_s(1^-) = D_s^\pm(2112)$ ,  $D_s(0^+) = D_{sJ}(2317)$ , and  $D_s(1^+) = D_{sJ}(2460)$  [10]. We expect these higher states of  $B$  and  $B_s$  mesons can be detected in CDF/LHC experiments by looking at their decay modes.

### III. CHIRAL LIMIT OF $H_0$

As mentioned above, the model by Bardeen and Hill [3] has chiral symmetry in the chiral limit of light quark mass,  $m_q \rightarrow 0$ . Our model has also chiral symmetry. Here let us see how it is realized.

In our model the lowest order mass of the  $Q\bar{q}$  bound state is given by  $m_Q + E_0^\ell$  after solving the following Schrödinger equation [9, 11],

$$H_0 \otimes \psi_0^\ell = E_0^\ell \psi_0^\ell, \quad H_0 = \vec{\alpha}_q \cdot \vec{p} + \beta_q (m_q + S(r)) + V(r), \quad (1)$$

where  $H_0$  is the lowest order Hamiltonian,  $\ell$  expresses all the quantum numbers,  $\psi_0^\ell$  is the lowest solution to Eq. (1), and with a notation  $\otimes$  one should understand that gamma matrices for a light antiquark be multiplied from left with the wave function while those for a heavy quark from right. Here  $S(r)$  is a confining scalar potential and  $V(r)$  is a Coulombic vector potential at short distances. Both potentials have dependency only on  $r$ , relative distance between  $Q$  and  $\bar{q}$ . Quantities with a subscript  $q$  mean those for a light antiquark. Using the spherical polynomials  $Y_j^m$  and the vector spherical harmonics defined by

$$\vec{Y}_{jm}^{(L)} = -\vec{n} Y_j^m, \quad \vec{Y}_{jm}^{(E)} = \frac{r}{\sqrt{j(j+1)}} \vec{\nabla} Y_j^m, \quad \vec{Y}_{jm}^{(M)} = -i\vec{n} \times \vec{Y}_{jm}^{(E)}, \quad (2)$$

we can decompose the wave function  $\psi_0^\ell$  into radial and angular parts as follows [9, 11];

$$\psi_0^\ell = \begin{pmatrix} 0 & \Psi_{jm}^k(\vec{r}) \end{pmatrix}, \quad \Psi_{jm}^k(\vec{r}) = \frac{1}{r} \begin{pmatrix} f_k(r) y_{jm}^k \\ ig_k(r) y_{jm}^{-k} \end{pmatrix}, \quad (3)$$

where the angular part  $y_{jm}^{\pm k}$  is given by the linear combination of  $Y_j^m$  and  $\vec{\sigma} \cdot \vec{Y}_{jm}^{(A)}$  ( $A=L, M, E$ ). Substituting this wave function into Eq. (1), one can obtain the radial part equation as follows;

$$\begin{pmatrix} m_q + S + V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & -m_q - S + V \end{pmatrix} \Psi_k(r) = E_0^k \Psi_k(r), \quad \Psi_k(r) \equiv \begin{pmatrix} f_k(r) \\ g_k(r) \end{pmatrix}. \quad (4)$$

Here  $k$  is the quantum number of the spinor operator  $K$  defined by [9, 11]

$$K = -\beta_q (\vec{\Sigma}_q \cdot \vec{L} + 1), \quad K \Psi_{jm}^k = k \Psi_{jm}^k. \quad (5)$$

where  $\vec{\Sigma}_q (= \vec{\sigma}_q \ 1_{2 \times 2})$  and  $\vec{L}$  are the 4-component spin and the orbital angular momentum of the light antiquark, respectively. Note that with this quantum number  $k$ , one can simultaneously determine both a partial angular momentum  $j_q$  of light degrees of freedom and a total parity  $P$  as [11],

$$j_q = |k| - \frac{1}{2}, \quad P = \frac{k}{|k|} (-)^{|k|+1}. \quad (6)$$

It is remarkable that in our approach  $K$  can be defined even for a two-body bound system composed of a heavy quark and a light antiquark.

Then, in our model a chiral limit is realized by setting  $m_q = S(r) = 0$ , in which the corresponding Hamiltonian becomes

$$H_0^{chiral} = \vec{\alpha}_q \cdot \vec{p} + V(r). \quad (7)$$

With this chiral Hamiltonian, the radial part equation becomes

$$\begin{pmatrix} V & -\partial_r + \frac{k}{r} \\ \partial_r + \frac{k}{r} & V \end{pmatrix} \Psi_k^{chiral}(r) = E_0^{k(chiral)} \Psi_k^{chiral}(r), \quad (8)$$

This equation can be converted into the one with  $-k$  by a unitary transformation with  $U = \sigma_2$ , where  $\sigma_2$  denotes the 2nd component of Pauli matrices,

$$\begin{pmatrix} V & -\partial_r - \frac{k}{r} \\ \partial_r - \frac{k}{r} & V \end{pmatrix} U \Psi_k^{chiral}(r) = E_0^{k(chiral)} U \Psi_k^{chiral}(r), \quad (9)$$

which means,

$$U \Psi_k^{chiral}(r) = \Psi_{-k}^{chiral}(r) \quad \text{and} \quad E_0^{k(chiral)} = E_0^{-k(chiral)}. \quad (10)$$

That is, the energy is degenerate in  $\pm k$  in the case of chiral Hamiltonian, Eq. (7), which corresponds to the degeneracy for  $k = \pm 1$  in the chiral limit. The degenerate hadron mass is given by  $m_Q$  since Eq.(8) is nothing but an equation for a hydrogen atom with mass = 0 and its energy level is proportional to mass which is zero in this case, i.e.,  $E_0^{k(chiral)} = 0$ .

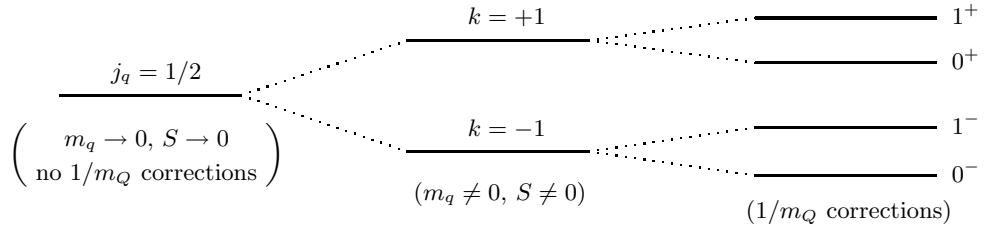
Considering the above observation, mass splitting in our model occurs as follows.

- (1) Start from the chiral limit Hamiltonian, Eq. (7), together with no  $1/m_Q$  corrections with  $m_Q$  heavy quark mass. In this stage, all the masses of  $0^-$ ,  $0^+$ ,  $1^-$  and  $1^+$  states with  $j_q = 1/2$  are degenerate;  $m_Q = m(0^-) = m(1^-) = m(0^+) = m(1^+)$ .
- (2) When the light quark mass  $m_q$  and a scalar potential  $S(r)$ , i.e. explicit chiral breaking terms are inserted as shown in Eq. (1), degeneracy is partially broken;  $m(0^-) = m(1^-)$  and  $m(0^+) = m(1^+)$ , which are called heavy quark multiplets in [2], because there still remains degeneracy due to the quantum number  $k$  [11].
- (3) Finally by including  $1/m_Q$  terms, all the degeneracy is resolved and the mass values in Tables I and II are given.

Notice that the quantum number  $k$  plays an important role in our model to see how the states are classified and how the degeneracy is resolved [11].

The above procedure may be depicted in Figure 1.

FIGURE 1: Procedure how the degeneracy is resolved in our potential model.



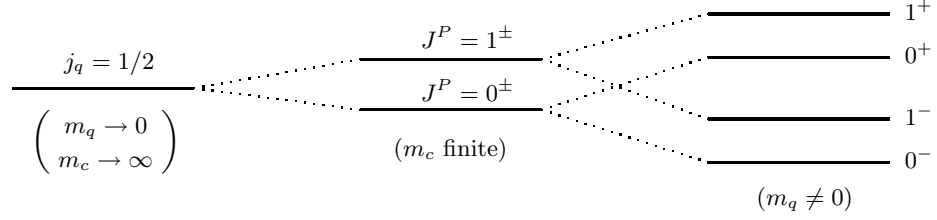
On the other hand, the procedure taken by Bardeen and Hill [3] can be stated as follows.

- (1) Start from the chiral limit of light quark masses but with infinite heavy quark mass instead of finite. In this stage, all the masses of  $0^-$ ,  $0^+$ ,  $1^-$ , and  $1^+$  states with  $j_q = 1/2$  are degenerate just like our model.
- (2) Inclusion of a finite heavy quark mass leads to a partial resolution of degeneracy;  $m(0^-) = m(0^+)$  and  $m(1^-) = m(1^+)$ , which are called parity doublets.
- (3) Finally by including a finite light quark mass effects and using the modified Goldberger-Treiman relation, all the degeneracy is resolved.

The above procedure may be depicted in Figure 2.

Comparing these two procedures, one can easily see that our model naturally explains how to resolve degeneracy between  $0^-$  and  $1^-$  and/or  $0^+$  and  $1^+$  due to the quantum number  $k$  and clarify the origin of mass splitting. Namely the interpretation by Bardeen, Eichten, and Hill is not the unique way to explain the mass splitting. There still remains a way to explain it using a potential model. Moreover, it should be noted that the potential model can give not only the mass differences of heavy mesons but also their absolute values, as seen in Tables I and II.

FIGURE 2: Procedure how the degeneracy is resolved in [2].



#### IV. CHECK OF THE GOLDBERGER-TREIMAN RELATION

Before closing this paper, we would like to give some comments on the modified Goldberger-Treiman relation claimed by Bardeen, Eichten, and Hill [2]. The modified Goldberger-Treiman relation predicts  $\Delta M(0) = \Delta M(1)$ , where

$$\Delta M(0) = M(0^+) - M(0^-), \quad \Delta M(1) = M(1^+) - M(1^-).$$

Then, one can test it by using the following mass differences for a couple of states.

(1)  $D_s(c\bar{s})$  states [1]

$$\begin{aligned} \Delta M(0) &= D_{sJ}(0^+, 2317) - D_s(0^-, 1969) = 348 \text{ MeV} \\ \Delta M(1) &= D_{sJ}(1^+, 2460) - D_s^*(1^-, 2112) = 348 \text{ MeV} \end{aligned}$$

(2)  $D(c\bar{u}(\bar{d}))$  states [7]

$$\begin{aligned} \Delta M(0) &= D(0^+, 2308) - D(0^-, 1870) = 438 \text{ MeV} \\ \Delta M(1) &= D(1^+, 2420) - D(1^-, 2010) = 410 \text{ MeV} \end{aligned}$$

(3)  $K(s\bar{u}(\bar{d}))$  states [10]

$$\begin{aligned} \Delta M(0) &= K(0^+, 1430) - K(0^-, 495) = 935 \text{ MeV} \\ \Delta M(1) &= K(1^+, 1335) - K(1^-, 895) = 440 \text{ MeV} \end{aligned}$$

where we have given mass of  $K(1^+) = 1335$  MeV as a simple average of  $K_1(1270)$  and  $K_1(1400)$ .  $D(0^+, 2308)$  is from the Belle data [7] and others are from the particle data group [10]. As one can see here, the modified Goldberger-Treiman relation looks to work amazingly well for  $D_s(c\bar{s})$  states. However, it might be problematic for other cases as shown in the following,

- (1) the mass of  $D(0^+)$  is expected to be 2280 MeV, if  $\Delta M(0) = \Delta M(1) = 410$  MeV holds, which is quite different from 2308 MeV observed by Belle [7]. Compared to this expectation, it is remarkable that in our case it is estimated to be 2304 MeV, as shown in Table I, being very close to the Belle data.
- (2) furthermore in the case of  $K$  states, it seems this relation cannot be applied, though its application to this case might not be justified because  $s$  quark is not heavy.

Based on this observation, the modified Goldberger-Treiman relation might be correct only in the case of  $D(c\bar{s})$  and there might be no more meaning for other states. To confirm this, it is very important to observe  $B(0^+)$  and  $B(1^+)$  as well as  $B_s(0^+)$  and  $B_s(1^+)$ , though a collective mass of orbitally excited ( $L = 1$ )  $B$  meson states was reported a couple of years ago [12]. We strongly wish that these particles will be observed soon.

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